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# SYSTEM AVAILABILITY AND RELIABILITYSUBJECT TO COMMON-CAUSE TIME-VARYING FUZZY RATES

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## ABSTRACT

This study presents a method for calculating the availability and reliability of a system depicted by block diagram, we use the Marshall and Olkin formulation of the multivariate exponential distribution. That is, the components are subject to failure by Poisson failure processes that govern simultaneous failure of a specific subset of the components. A model is proposed for the analysis of systems subject to common-cause time to simultaneous failure and the time to repair of each state follow Rayleigh distribution with unknown parameters which can be represented by triangular fuzzy numbers estimated using the statistical data then we introduce the procedures to determine the availability function, the reliability function. The method for calculating the system availability and reliability requires that a procedure exists for determining the system availability and reliability from component availabilities and reliabilities, under the statistically independent component assumption. A numerical example has been studied in detail to illustrate the model and to get analytic and graphical results.

**KEYWORDS:** System Availability, System Reliability, Common-Cause Failures, Fuzzy Rayleigh Distribution

## 1. INTRODUCTION

COMMOM CAUSE FAILURE (CCF) is the failure of multiple components due to a CC (single occurrence or condition). The origin of CC events can be outside the system elements they affect (e.g., lightning events that cause outages of unprotected electronic equipment) or can originate from the elements themselves, causing other elements to fail (e.g., voltage surges caused by inappropriate switching in power systems that lead to failure propagation). CCF increase joint-failure probabilities, thereby reducing the reliability of technical systems. Several papers have been devoted to modeling CCF distributions[1]–[3] and estimating the effect of CCF on system reliability or availability [4]–[12]. There are two approaches for incorporating CCF into system reliability analysis: explicit and implicit [7].

Fuzzy set was introduced firstly by Zadah [13] then it was applied in various fields containing uncertainty as Markov chains (Buckley [14]).

For the real time conditions, Chen [15] presented a new method for system reliability analysis based on the α-cuts arithmetic operation on the fuzzy time series. Wang [16] applied fuzzy random lifetimes for a series and parallel system, and Sharifi suggested an algorithm for reliability evaluation of a system containing n elements connected in parallel as in [17] or in a k-out-of-n system [18] assuming the failure rates are increasable and represented by fuzzy numbers.

El-Damcese and Temraz [19] use a model for a k-out-of-n: F system that consists of n independent and identical components connected in parallel using non-homogeneous/ homogeneous continuous-time Markov chain.

**Notation**

**n:** number of components in the system;

**k:** number of good components that allow the system to operate;

**$Z_r$:** Poisson failure process that governs the simultaneous failure of a specific set of *r* components;

**$S_i$:** Event that component i is good;

: number of combinations of *r* items out of a possible n items; $\binom{n}{r}$

$p_n^k(t)$ :probability that all components of a specific k-component subset out of an n-component system are operating at time t;

$A_{CC}(t)$ : system availability at time *t* with identically distributed components having common-cause failures;

$A_{SC}(t)$ : system availability at time *t* with i.i.d. components;

$h_i(t)$:(failure/ repair) rate of component i;

$\theta_i$ , $\alpha_i$ : the parameter of the (failure/ repair)rate distributionof component i;

$R_{CC}(t)$ reliability at time *t* with identically distributed components having common-cause failures;

$R_{SC}(t)$ : reliability at time *t* with i.i.d. components;

i.i.d.: s-independent and identically distributed.

## 2. COMPONENT AVAILABILITY AND RELIABILITYMODEL

Figure 1 is the state transition diagram for the 1-component availabilitymodel.

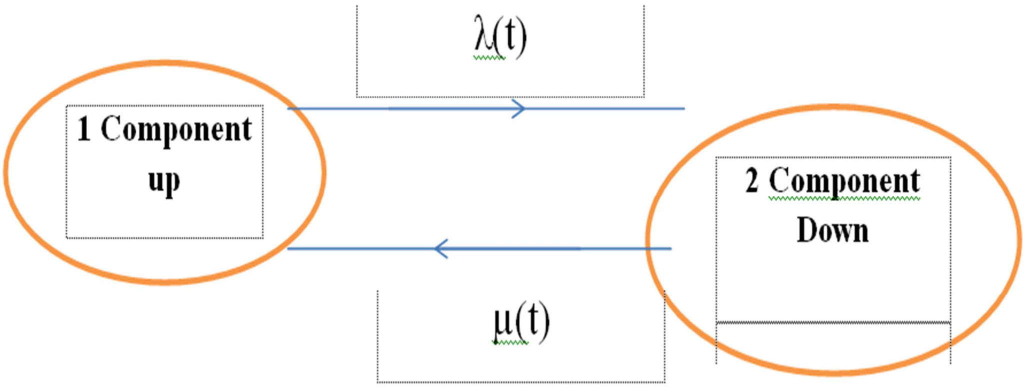


**Figure 1: Component Availability State- Transition Diagram**

Probability of working state 1 at time t is:

$$\frac{dP_1(t)}{dt} = -\lambda(t)P_1(t) + \mu(t)P_2(t)$$

since $P_1(t)+P_2(t)=1$, we have

$$\frac{dP_1(t)}{dt} = -[\lambda(t)+\mu(t)]P_1(t) + \mu(t) \tag{1}$$

Let the (failure/repair) function of a component following a 1-parameter Rayleigh distribution can be described by:

$$\lambda(t) = \frac{t}{\theta^2}, \quad \mu(t) = \frac{t}{\alpha^2} \tag{2}$$

thus,

$$\frac{dp_1(t)}{dt} = -[\frac{t}{\theta^2} + \frac{t}{\alpha^2}]p_1(t) + \frac{t}{\alpha^2}$$

since $P_1(0)=1$, then the availability A(t) of component is

$$A(t) = p_1(t) = \frac{1}{\theta^2+\alpha^2}[\theta^2 + \alpha^2 \exp[-\frac{1}{2}(\frac{\theta^2+\alpha^2}{\alpha^2\theta^2})t^2]] \tag{3}$$

In general, the availability of component i is:

$$A_i(t) = \frac{1}{\theta_i^2+\alpha_i^2}[\theta_i^2 + \alpha_i^2 \exp[-\frac{1}{2}(\frac{\theta_i^2+\alpha_i^2}{\theta_i^2\alpha_i^2})t^2]] \text{, i=1, 2, …, n} \tag{4}$$

In special case for without repair, the reliability of component i is:

$$R_i(t) = \exp[-\frac{t^2}{2\theta_i^2}] \text{, i=1, 2, …, n} \tag{5}$$

## 3. SYSTEM AVAILABILITY AND RELIABILITY ANALYSISWITH COMMON-CAUSE HAZARDS

A specific component can fail due to the occurrence of several different failure processes.

1. There is the 1-componentprocess $Z_1$for s-independent failure of thespecified component.

2. There are 2-component processesthat include the specified component.There are a total of$\binom{n}{2}$ i.i.d. $Z_2$failure processes but only ,$\binom{n-1}{1}$of these processes include the specified component. In general,there are$\binom{n}{r}$ i.i.d. $Z_r$failure processes with exponential parameters, governing thesimultaneous failure of *r* components. Of these$\binom{n}{r}$failure processes,$\binom{n-1}{r-1}$of them include the specified component.

$A_n^{(1)}(t)$ is the probability that the specified component is operating at time t, viz, the probability that none ofthe processes governing the simultaneous failure of *r*components,*r* == 1,2, ...,*n* includes the specific component. Based on s-independence of the Poisson processes-

$$A_n^{(1)}(t) = \prod_{i=1}^{n} [A_i(t)]^{\binom{n-1}{i-1}} \tag{6}$$

The probability that a specific group of kcomponents out of n-component system are all good is:

$$A_n^{(k)}(t) = \Pr\{S_1 \cap S_2 \cap \ldots\ldots \cap S_k ;t\}.$$

$$A_n^{(k)}(t) = \Pr\{S_1;t\}\Pr\{S_2/S_1;t\}\ldots\ldots\Pr\{S_k/S_1,S_2,\ldots\ldots,S_{k-1};t\}$$

$$A_n^{(k)}(t) = A_n^{(1)}(t)A_{n-1}^{(1)}(t)\ldots.A_{n-k+1}^{(1)}(t)$$

thus,

$$A_n^{(k)}(t) = \prod_{m=n-k+1}^{n} A_m^{(1)}(t) \tag{7}$$

These formulas were originally derived fromKyung for constant failure rates; similar argumentsare valid for time-varying failure rates.

The results are $A_{CC}(t)$ and $A_{SC}(t)$ in terms ofavailabilities $A_i(t)$ .

$R_n^{(1)}(t)$ is the probability that the specified component is operating at time t without repair, Based on the s-independence of the Poisson processes, we have:

$$\mathrm{R}_\mathrm{n}^{(1)}(\mathrm{t}) = [\textstyle\prod_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{R}_\mathrm{i}(\mathrm{t})]^{\binom{\mathrm{n}-1}{\mathrm{i}-1}} \tag{8}$$

The probability that a specific group of k components without repair out of n-component system are all good is:

$$R_n^{(k)}(t) = \prod_{m=n-k+1}^{n} R_m^{(1)}(t) \tag{9}$$

**4. THE FUZZY SYSTEM RELIABILITY AND AVAILABILITY**

Due to uncertainty in the values of parameters, they can be modeled by triangular fuzzy number, we use the triangular membership function:$\tilde{\theta}(L_i , M_{i,} U_i)$,$\tilde{\alpha}(L_i^{'} , M_i^{'} , U_i^{'})$

we can represent fuzzy failure and repair rates by crisp intervals using α-cuts of membership functions as follows:

$$= [L_i + \alpha(M_i - L_i), U_i - \alpha(U_i - M_i)], 0 \le \alpha \le 1 \tag{10}$$

$$[\tilde{\alpha}_i{}^L, \tilde{\alpha}_i{}^U]_{\alpha-cut} = [L_i' + \tilde{\alpha}(M_i' - L_i'), U_i' - \alpha(U_i' - M_i')], 0 \leq \alpha \leq 1 \quad (11)$$

Where $M_i$ ($M_i'$), $L_i$ ($L_i'$)and $U_i$ ($U_i'$) are the point estimation, lower and upper of $\tilde{\theta}_i, \tilde{\alpha}_i$ respectively

In general, if m, the size of random sample, then the point estimation and the $(1-\gamma)100\%$ confidence interval for each parameter $\tilde{\theta}_i, \tilde{\alpha}_i$can be calculated from the following relations.

$$M = \sqrt{\sum_{i=1}^{m} X_i{}^2/2m}, M' = \sqrt{\sum_{i=1}^{m'} X_i{}^2/2m'} \quad (12.1)$$

$$[L, U] = \left[M \pm Z_{\gamma/2}\sqrt{var(M)}\right], [L', U'] = \left[M' \pm Z_{\gamma/2}\sqrt{var(M')}\right] \quad (12.2)$$

Where, $var(M) = \frac{M^2}{4m}, var(M') = \frac{M'^2}{4m'}$,$\gamma$ =0.05 (12.3)

## 5. ILLUSTRATIVE EXAMPLE

The system in Figure 1 consisting of 10- components in two subsystems A, B arranged in series-parallel. Subsystem A consist of two paths each contains two components $A_i$ , i=1, 2. The two paths are parallel while subsystem B consists of two paths each contains three components $B_i$ , i=1, 2 arranged in series. However the two paths are parallel to each other. The system failed when any of the two subsystem A or B failed.

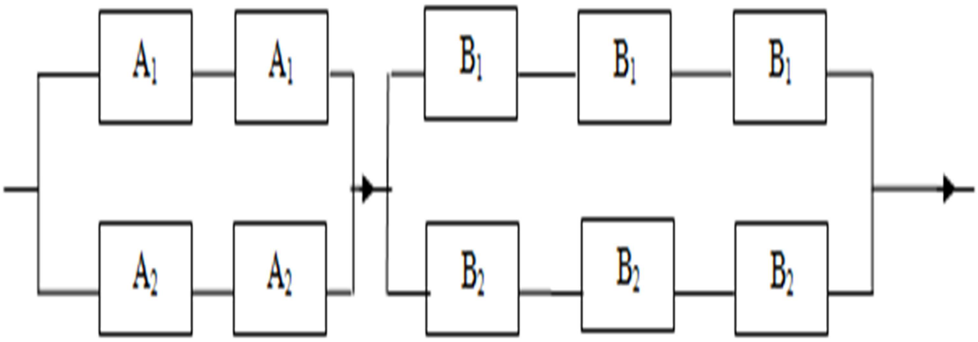


**Figure 2: Block Diagram of System**

For identically distributed components with statistically-independent failure processes, the availability $A_{SC}$ (t) of the whole system can then beevaluated as:

$$A_{SC}(t) = 4A^5(t) - 2A^7(t) - 2A^8(t) + A^{10}(t) \quad (13)$$

Substituting (4) in (13) for$\theta_i$=$\theta$=2.600 and $\alpha_i$=$\alpha$=1.800 , i=1,2,..,10, the availability $A_{SC}$(t) for this system against time t is shown inFigure3.

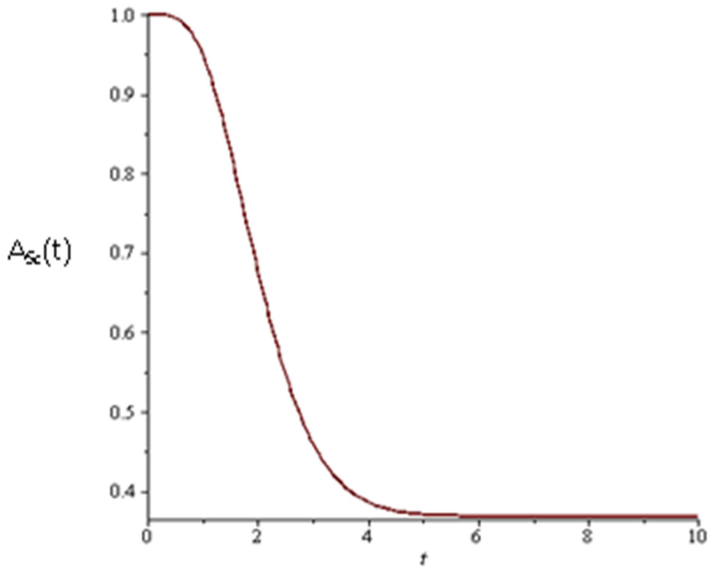


**Figure 3: System Availability for i.i.d. Components**

For identically distributed components with statistically-independent failure processes, the reliability $R_{SC}$ (t) of the whole system with associated equation (5) when$\theta_i=\theta$, i=1,2,..,10, can then beevaluated as:

$$R_{SC}(t)=4\exp[-\frac{5t^2}{2\theta^2}]-2\exp[-\frac{7t^2}{2\theta^2}]-2\exp[-\frac{4t^2}{\theta^2}]+\exp[-\frac{5t^2}{\theta^2}] \tag{14}$$

Now for θ=2.600we can use the previous equation to study the effect of increasing time t on reliability $R_{SC}(t)$ for this system in the following Figure.

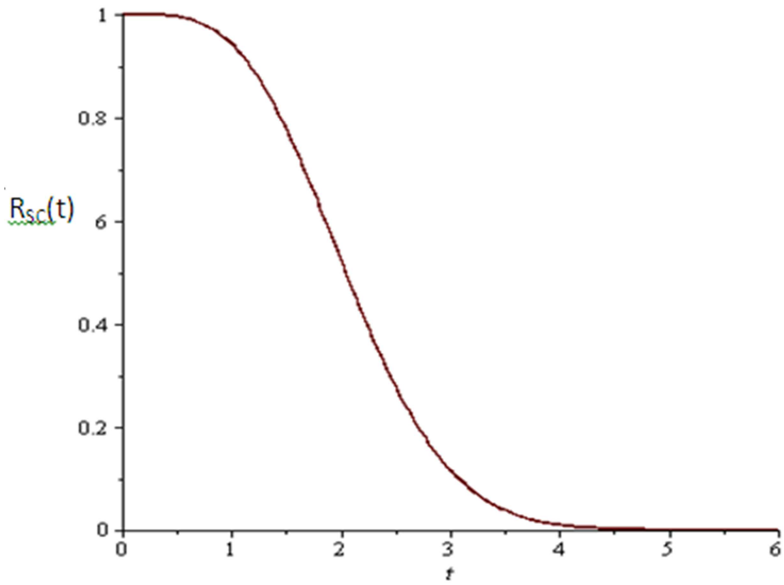


**Figure 4: System Reliability for i.i.d. Components**

For comparison purposes, the one-component availability remains at the value of a component in the ten-component common-cause system, but the system consists statistically-independent and identically distributed (i.i.d) components that are, when $A(t) = A_{10}^{(1)}(t)$ in equation (13) the system effects of common-cause failures and represents the prediction of a practitioner assessing all failures causes against a component, but assuming a "statistically-independence" model. In that case, we have:

$$A_{10}^{(1)}(t) = \prod_{i=1}^{10} [A_i(t)]^{\binom{9}{i-1}}$$

$$= A_1(t) A_2^9(t) A_3^{36}(t) A_4^{84}(t) A_5^{126}(t) A_6^{126}(t) A_7^{84}(t) A_8^{36}(t) A_9^9(t) A_{10}(t)$$

When the identically distributed components have common-cause failures, we have:

$$A_{CC}(t) = 4A_{10}^{(5)}(t) - 2A_{10}^{(7)}(t) - 2A_{10}^{(8)}(t) + A_{10}^{(10)}(t) \tag{15}$$

Where:

$$A_{10}^{(k)}(t) = \prod_{m=11-k}^{10} A_m^{(1)}(t) \quad , k = 5,7,8,10$$

Let the failure and repair rates are:

The Parameters $\theta_i$ and $\alpha_i$, assuming failure and repair rates for number of simultaneous failures

| Failure Parameter | Number of Simultaneous Failures | Simultaneous Failure Rate | Repair Parameter | Simultaneous Repair Rate |
|---|---|---|---|---|
| $\theta_1$, $\theta_2$ | 1, 2 | $\lambda_I(t)$ | $\alpha_1$, $\alpha_2$ | $\mu_I(t)$ |
| $\theta_3$, $\theta_4$ | 3, 4 | $\lambda_{II}(t)$ | $\alpha_3$, $\alpha_4$ | $\mu_{II}(t)$ |
| $\theta_5$, $\theta_6$ , $\theta_7$ | 5, 6, 7 | $\lambda_{III}(t)$ | $\alpha_5$, $\alpha_6$ , $\alpha_7$ | $\mu_{III}(t)$ |
| $\theta_8$, $\theta_9$, $\theta_{10}$ | 8, 9, 10 | $\lambda_{IV}(t)$ | $\alpha_8$, $\alpha_9$, $\alpha_{10}$ | $\mu_{IV}(t)$ |

In that case, we have:

$$A_{10}^{(5)}(t) = \prod_{m=6}^{10} A_m^{(1)}(t) = A_6^{(1)}(t) A_7^{(1)}(t) A_8^{(1)}(t) A_9^{(1)}(t) A_{10}^{(1)}(t)$$

$$= A_I^{40}(t) A_{II}^{315}(t) A_{III}^{581}(t) A_{IV}^{56}(t)$$

$$A_{10}^{(7)}(t) = \prod_{m=4}^{10} A_m^{(1)}(t) = A_4^{(1)} A_5^{(1)} A_6^{(1)}(t) A_7^{(1)}(t) A_8^{(1)}(t) A_9^{(1)}(t) A_{10}^{(1)}(t)$$

$$= A_I^{49}(t) A_{II}^{329}(t) A_{III}^{582}(t) A_{IV}^{56}(t)$$

$$A_{10}^{(8)}(t)=\prod_{m=3}^{10}A_m^{(1)}(t)=A_3^{(1)}A_4^{(1)}A_5^{(1)}A_6^{(1)}(t)A_7^{(1)}(t)A_8^{(1)}(t)A_9^{(1)}(t)A_{10}^{(1)}(t)$$
$$=A_I^{52}(t)A_{II}^{330}(t)A_{III}^{582}(t)A_{IV}^{56}(t)$$

$$A_{10}^{(10)}(t)=\prod_{m=1}^{10}A_m^{(1)}(t)=A_1^{(1)}A_2^{(1)}A_3^{(1)}A_4^{(1)}A_5^{(1)}A_6^{(1)}(t)A_7^{(1)}(t)A_8^{(1)}(t)A_9^{(1)}(t)A_{10}^{(1)}(t)$$
$$=A_I^{55}(t)A_{II}^{330}(t)A_{III}^{582}(t)A_{IV}^{56}(t)$$

By substituting in equation (15) we find $A_{CC}(t)$ where $\lambda_i(t)$, $\mu_i(t)$ are given by

| Number of Simultaneous of Failure and Repair Rates | $\lambda_i(t)$ | $\mu_i(t)$ |
|---|---|---|
| I | $t/(2.600)^2$ | $t/(2.100)^2$ |
| II | $t/(2.700)^2$ | $t/(1.800)^2$ |
| III | $t/(2.750)^2$ | $t/(1.700)^2$ |
| IV | $t/(2.760)^2$ | $t/(1.600)^2$ |

The availability $A_{CC}(t)$ for this system against time t is shown inFigure5.

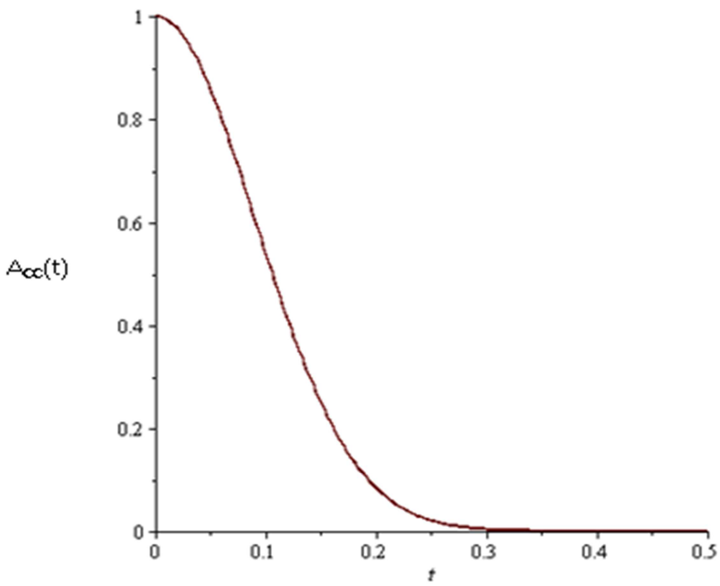


**Figure 5: System Availability for Common Cause Components**

At time 0.2 the i.i.d. system availability becomes:

$A_{SC}$ (0.2)= 0.9998, $A_{CC}$ (0.2)= 0.0840 thus, for this case the system availability assuming common-cause failures is lower than the i.i.d. system availability.

For comparison purposes, the one-component reliability remains at the value of a component in the 10-component common-cause system, but the system consists statistically-independent and identically distributed (i.i.d) components that are, calculate $R_{SC}$ (t) when $R(t)=R_{10}^{(1)}(t)$ in equation (14).

The resulting reliability neglects, the system effects of common-cause failures and represents the prediction of a practitioner assessing all failures causes against a component, but assuming a "statistically independence" model. In that case, we have:

$$R_{10}^{(1)}(t)=\prod_{i=1}^{10}[\exp[-\frac{t^2}{2\theta_i^2}]]^{\binom{9}{i-1}} \tag{16}$$

When the identically distributed components have common-cause failures, we have:

$$R_{CC}(t) = 4R_{10}^{(5)}(t) - 2R_{10}^{(7)}(t) - 2R_{10}^{(8)}(t) + R_{10}^{(10)}(t) \tag{17}$$

where

$$R_{10}^{(k)}(t)=\prod_{m=11-k}^{10} R_m^{(1)}(t), \quad \mathrm{k}=5,7,8,10$$

Where the reliability of a single component in a 10-component system given by(16) is:

$$R_{10}^{(1)}(t)=\exp[-(\frac{1}{\theta_I^2}+\frac{9}{\theta_I^2}+\frac{36}{\theta_{II}^2}+\frac{84}{\theta_{II}^2}+2\times\frac{126}{\theta_{III}^2}+\frac{84}{\theta_{III}^2}+\frac{36}{\theta_{IV}^2}+\frac{9}{\theta_{IV}^2}+\frac{1}{\theta_{IV}^2})(\frac{t^2}{2})]$$

We find $R_{10}^{(5)}(t), R_{10}^{(7)}(t), R_{10}^{(8)}(t)\ and\ R_{10}^{(10)}(t)$ similar as availability and substitution in equation (17) we find $R_{CC}(t)$ by using previous values of the simultaneous of failure rates.

The reliability $R_{CC}$ (t) for this system against time t is shown in Figure 6.

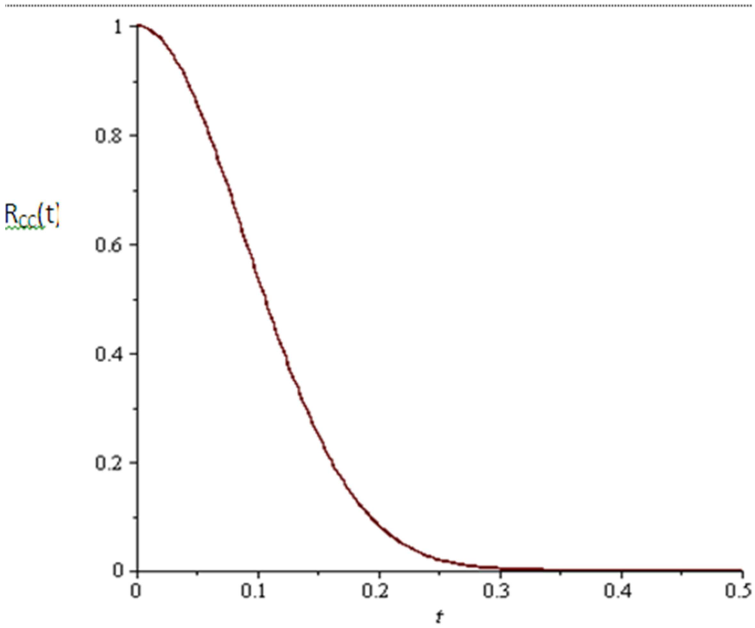


**Figure 6: System Reliability for Common Cause Components**

At time 0.2 the i.i.d. system reliability becomes:

$R_{SC}(0.2) = 0.9998$, $R_{CC}(0.2)= 0.0833$ thus, for this case the system reliability assuming common-cause failures is lower than the i.i.d. system reliability.

Consider that the life and repair times follow Rayleigh distribution with fuzzy parameters, so the (failure/repair) rates are given by the following relation:

fuzzy(failure/repair) rates are:

$$\tilde{\lambda}_i(t) = \frac{t}{\tilde{\theta}_i^2}, \quad \tilde{\mu}_i(t) = \frac{t}{\tilde{\alpha}_i^2}, \quad i = I, II, III, IV$$

Thus,

$$\tilde{A}_i(t) = \frac{1}{\tilde{\theta}_i^2 + \tilde{\alpha}_i^2}[\tilde{\theta}_i^2 + \tilde{\alpha}_i^2 \exp - \frac{1}{2}(\frac{\tilde{\theta}_i^2 + \tilde{\alpha}_i^2}{\tilde{\theta}_i^2 \tilde{\alpha}_i^2})t^2], \quad i = I, II, III, IV$$

$$\tilde{R}_i(t) = \exp[-\frac{t}{2\tilde{\theta}_i^2}], \quad i = I, II, III, IV$$

Now, we will apply the introduced procedure with setting the following data.

**For** $\tilde{\theta}_I, \tilde{\alpha}_I$ **:** Let $m = 70, \sum_{i=1}^{70} X_i^2 = 1220, m' = 70, \sum_{i=1}^{70} {X'}_i^2 = 800$,

**For** $\tilde{\theta}_{II}, \tilde{\alpha}_{II}$ **:** Let $m = 50, \sum_{i=1}^{50} X_i^2 = 1000, m' = 50, \sum_{i=1}^{50} {X'}_i^2 = 600$,

**For** $\tilde{\theta}_{III}, \tilde{\alpha}_{III}$ **:** Let $m = 40, \sum_{i=1}^{40} X_i^2 = 850, m' = 40, \sum_{i=1}^{40} {X'}_i^2 = 400$,

**For** $\tilde{\theta}_{IV}, \tilde{\alpha}_{IV}$ **:** Let $m = 36, \sum_{i=1}^{36} X_i^2 = 800, m' = 36, \sum_{i=1}^{36} {X'}_i^2 = 250$.

We calculate the intervals for the parameters $\tilde{\theta}_i, \tilde{\alpha}_i$, i=I, II, III, IV corresponding to the α-cuts and the results are show in tables 1 and 2, respectively.

**Table 1: The Intervals for $\tilde{\theta}_I, \tilde{\theta}_{II}, , \tilde{\theta}_{III}, \tilde{\theta}_{IV}$ Corresponding to *α-cut* = 0, 0.1, 0.2, 0.3, 0.4, 0.5**

| α-cut | $[\tilde{\theta}_I^L, \tilde{\theta}_I^U]$ | $[\tilde{\theta}_{II}^L, \tilde{\theta}_{II}^U]$ | $[\tilde{\theta}_{III}^L, \tilde{\theta}_{III}^U]$ | $[\tilde{\theta}_{IV}^L, \tilde{\theta}_{IV}^U]$ |
|---|---|---|---|---|
| 0 | [2.606,3.295] | [2.724,3.599] | [2.757,3.760] | [2.787,3.872] |
| 0.1 | [2.640,3.260] | [2.767,3.555] | 2.807,3.709] | [2.841,3.817] |
| 0.2 | [2.675,3.226] | [3.811,3.511] | [2.857,3.659] | [2.895,3.763] |
| 0.3 | [2.709,3.191] | [2.855,3.467] | [2.907,3.609] | [2.907,3.609] |
| 0.4 | [2.744,3.157] | [2.899,3.424] | [2.957,3.559] | [2.957,3.559] |
| 0.5 | [2.778,3.123] | [2.943,3.380] | [3.008,3.509] | [3.008,3.509] |

**Table 2: The Intervals for $\widetilde{\propto}_I, \widetilde{\propto}_{II}, , \widetilde{\propto}_{III}, \widetilde{\propto}_{IV}$ Corresponding to *α-cut* = 0, 0.1, 0.2, 0.3, 0.4, 0.5**

| α-cut | $[\widetilde{\propto}_I{}^L, \widetilde{\propto}_I{}^U]$ | $[\widetilde{\propto}_{II}{}^L, \widetilde{\propto}_{II}{}^U]$ | $[\widetilde{\propto}_{III}{}^L, \widetilde{\propto}_{III}{}^U]$ | $[\widetilde{\propto}_{IV}{}^L, \widetilde{\propto}_{IV}{}^U]$ |
|---|---|---|---|---|
| 0 | [2.113,2.666] | [2.109,2.788] | [1.891,2.580] | [1.561,2.164] |
| 0.1 | [2.140,2.638] | [2.143,2.754] | [1.925,2.545] | [1.591,2.133] |
| 0.2 | [2.168,2.610] | [2.177,2.720] | [1.96,2.511] | [1.621,2.103] |
| 0.3 | [2.196,2.583] | [2.211,2.686] | [1.994,2.476] | [1.651,2.073] |
| 0.4 | [2.223,2.555] | [2.245,2.652] | [2..029,2.442] | [1.681,2.043] |
| 0.5 | [2.251,2.528] | [2.279,2.618] | [2.063,2.408] | [1.712,2.013] |

Using MAPLE programme we can calculate the availability functions$\tilde{A}_{SC}(t), \tilde{A}_{CC}(t)$ and the reliability functions$\tilde{R}_{SC}(t), \tilde{R}_{CC}(t)$ . We get the fuzzy availability and reliability functionsand represent them graphically at different values of α-cut=0, 0.3, 0.5 are show in Figures 7-10 .

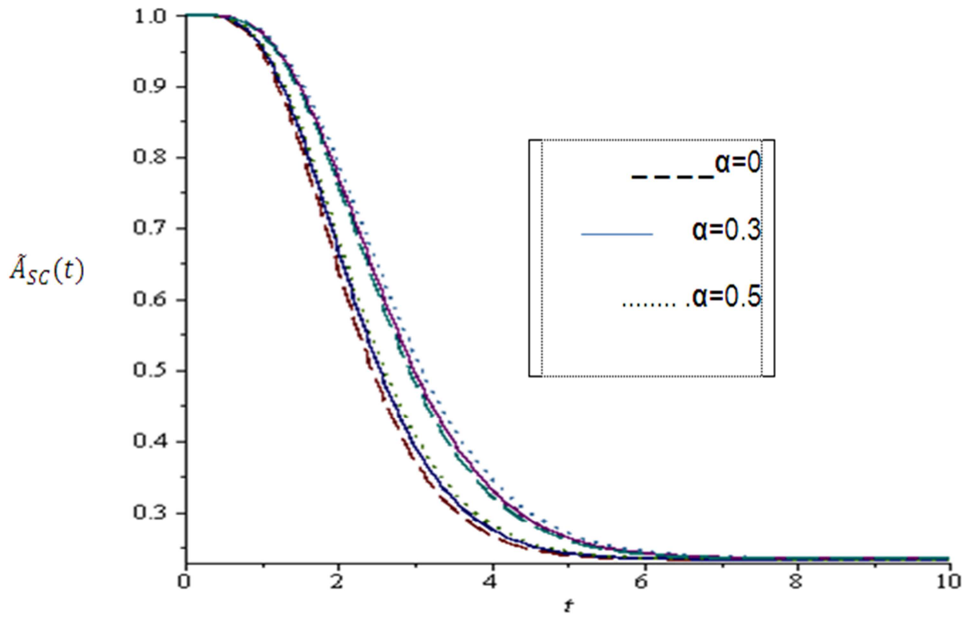


**Figure 7: System Availa!Bility for i.i.d. Components$\tilde{A}_{SC}(t)$**

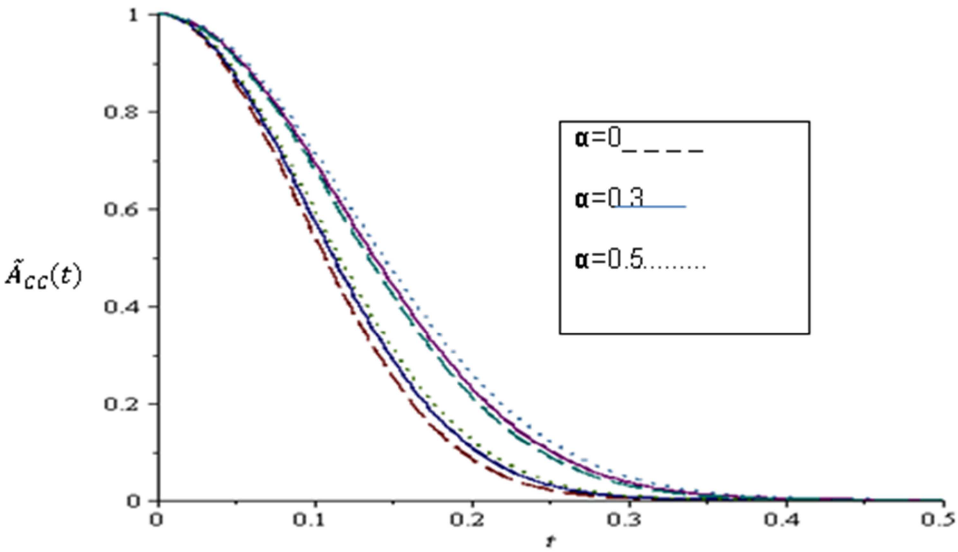


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**Figure 8: System Availability for Common-Cause Components $\tilde{A}_{CC}(t)$**

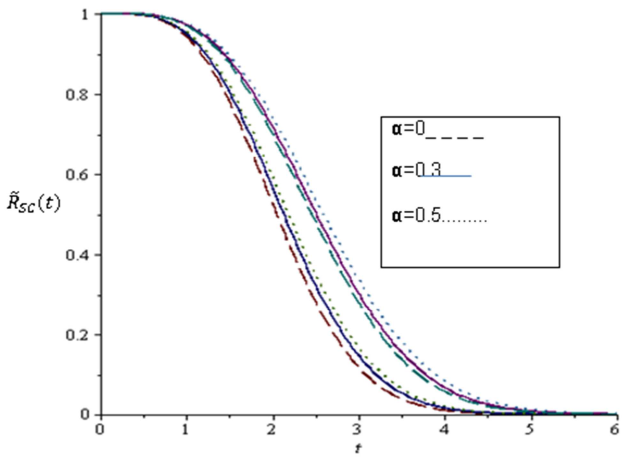


**Figure 9: System Reliability for i.i.d. Components$\widetilde{R}_{SC}(t)$**

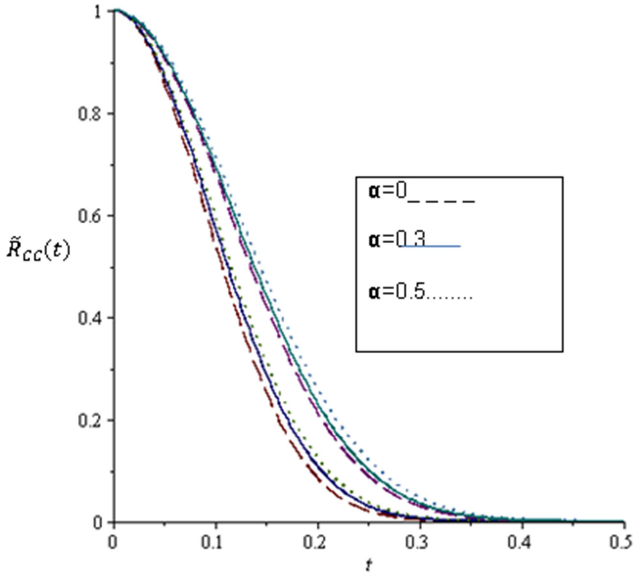


**Figure 10: System Reliability for Common-Cause Components $\widetilde{R}_{CC}(t)$**

## 6. CONCLUSIONS

In this paper, we proposed Rayleigh distribution to analyze the i.i.d. and CCF of the systems reliability and availability. the result shows that the systems availability and reliability , assuming common-cause, failures, is appreciably lower than the i.i.d. systems availability and reliability. In this paper the parameter was considered as fuzzy triangular number and their α-cut set are presented. Also, we obtained the numerical solutions of the system consisting of 10-components in two subsystems A, B arranged in series-parallel.

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